

well informed about recent scholarship in the history of Iranian religion, where he advances some highly risky conjectures, nor is he wary about the credibility of late Greek accounts of the pre-Socratics or of Oriental beliefs (and he cites only those sources, ancient and modern, that support his thesis). In addition, he believes that laws of symmetry and simplicity are as applicable to history as they are to mathematics. The result is an extremely unsophisticated and unpersuasive simplification of the very complex interactions of different cultures that eventually produced deterministic astrology and an associated astral religion in the last two centuries B.C.

These faults in the author's conception of historical methodology are not confined to his treatment of the history of astrology and astral religion. In his own field of the history of astronomy he succumbs all too often to the need to have an answer even when there is no convincing evidence, as in his claim that the Babylonian System A for the planets was invented by Nabu Rimannu in about -500 (p. 283); or even when the evidence is contrary to his conclusion, as when he claims that Rhethorius' Great Year was based on the parameters of Babylonian astronomy (p. 113) (in fact, Rhethorius' Great Year--1,735,200 years is the correct reading rather than van der Waerden's 1,735,005--is 1,200 Sothic periods, and therefore based on an Egyptian parameter.)

This book will probably be used in classrooms in American colleges. Unfortunately the students who are asked to read it will in general have no way of distinguishing what in it is plausible as history and what is not.

L'ARITHMÉTIQUE AMUSANTE. [Arithmetical recreations]. By Edouard Lucas. Facsimile reprint. Paris (Albert Blanchard). 1974. 266 p. fr. 22.

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Edouard Lucas died in 1891 at the height of his career when only forty-nine. At the expressed wish of the family, the Société Mathématique de France designated a Commission composed of H. Delannoy, C.-A. Laisant, and E. Lemoine to complete the publication of all that Lucas had written on the subject. The commission discovered the unedited manuscripts of Volumes 3 and 4 of the *Récréations mathématiques*, which they promptly published in 1893 and 1894. Lucas had planned to publish Volumes 5 and 6, but only the titles of the fifteen projected chapters were ever found.

Among his papers the Commission also found the manuscript

of *L'Arithmétique amusante*, which was then published in 1895. It is written in the same droll style that characterizes the *Récréations mathématiques*. The first two chapters deal with *Calculs élémentaires* and *Le calcul rapide*, respectively. The former discusses a solution of the Josephus problem (15 Turks and 15 Christians) by using playing cards. For the rest, there are number tricks and puzzles, some domino diversions, and a 3 x 3 magic square formed with the playing cards Ace-through 9. The second chapter, stressing mental arithmetic, deals with further "number pleasantries" such as complementary numbers, multiplication by 9 and 11; short cuts for multiplication such as 46 x 38; Napier's bones; shortcuts for division by 19, 199, 1999, and by 29, 299, 2999; various ways of expressing the number 9 using each of the ten digits once, and the number 100, using the digits 1-9 each once; e.g. $9 = \frac{97524}{10836}$, and $94 \frac{1578}{263} = 100$. The chapter closes with a lengthy exposition of the card game known as *Rouges et Noires*.

Chapter 3, entitled *Les progressions arithmétiques*, deals with familiar recreational problems such as the number of times a clock chimes; finding the distance run in retrieving apples or potatoes arranged at equal intervals in a row; and a variety of ferrying problems, or "difficult crossings", notably about jealous husbands and their wives crossing a stream in a boat too small to hold them all. The discussion includes situations involving 2, 3, 4 and 5 couples (with varying restrictions), and concludes with a general case. In passing, Lucas points out an incorrect solution attributed to Tartaglia.

The final chapter, *Les progressions géométriques*, deals with some well known recreations, among them the story of the number of grains of wheat upon a checkerboard; the amazing result if one centime had been left at 5% compound interest since the birth of Christ; Archimedes' scheme for determining the number of grains of sand in the known world. Other topics include binary notation and its relation to the Chinese book of combinatorics (Je-Kim); the problem of the minimum number of different weights required to make all integral weighings up to a stated amount; the ternary scale of notation; the Chinese ring puzzle (Le Bagenaudier); the Tower of Hanoi; and the problem of removing 1 liter of wine from a 100-liter full casket of wine replacing it with a liter of water, repeating the operation 19 times, and deciding how much wine remains after 20 such operations.

Interesting material is also to be found in the notes following Chapter 4 (pp. 187-260). NOTE I is an informal sketch of numeration and computation, including Genaille's device for multiplication. NOTE II is a discussion of certain variations of the difficult crossings problem, notably where one of the jealous husbands is a polygamist with two wives. NOTE III deals briefly with six games attributed to Lucas and which were exhibited

at the Exposition Universelle of 1889, ostensibly to enhance Lucas' reputation. The games are identified as 1) *La Faisioulette* (*courses du cavalier sur l'échiquier*); 2) *La Pipopipette* (also known as *Jeu de l'Ecole Polytechnique*); 3) *La Tour d'Hanoi* (Tower of Hanoi); 4) *L'Icosagonal* (*Jeu de vingts forts*); 5) *L'Arithmétique diabolique*; 6) *Les Pavés florentins du père Sébastien*. La Pipopipette is a pencil and paper game created by students of the Ecole Polytechnique, played on quadrille paper upon which squares within squares have been drawn.

NOTE IV deals with miscellaneous recreations: (1) the problem of the eight queens; (2) *Les Jeux du Rubans*, i.e., folding a ribbon of paper to form a regular pentagon; a magic square of order 9 involving the first 81 integers, accompanied by a curious Latin inscription discovered in Rome in 1881; (4) a magic 4 x 4 cube involving the first 64 integers, due to Fermat; (5) bordered magic squares, also due to Fermat; and (6) various solutions of the problem of the *cavalier au jeu des échecs* and related problems as given by Ciccolini, Jaenisch, Vandermonde, Warnsdorf, F. Sainte-Marie and others.

To conclude: *L'Arithmétique amusante* may be described as a preparation for, or an introduction, to the four-volume work for which Lucas is justly remembered.

FUNCTIONAL ANALYSIS IN HISTORICAL PERSPECTIVE. By A. F. Monna.
Utrecht (Oosthoek Publishing Co). 1973. 175 p.

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Ce petit livre a été écrit pour montrer que la plus grande partie de l'analyse moderne a son origine dans des problèmes de mathématiques classiques (physique, mécanique, probabilités). Il ne veut en aucun cas donner une description encyclopédique de cette évolution, qu'on peut trouver, par exemple, dans les travaux de M. Bernkopf [1966, 1968], mais son but est double:

- donner les grandes lignes de ce développement,
- mettre en lumière les travaux de pionniers qui restent encore de nos jours injustement ignorés.

On y découvre un certain nombre de faits assez surprenants: d'abord que la naissance de l'algèbre linéaire axiomatique n'a pratiquement pas influé sur celle de l'analyse fonctionnelle, ensuite que cette dernière est née sous la forme de deux branches jumelles qui n'eurent aucuns rapports entre elles, la première constituée par l'école allemande--renforcée par les mathématiciens hongrois et polonais--, la seconde par l'école italienne.

Le premier chapitre rappelle d'abord les questions classiques qui amenèrent Hill, Poincaré, von Koch et Hilbert, au début du siècle, à la résolution de systèmes infinis à une infinité